Experimental control of single-mode laser chaos by using continuous, time-delayed feedback

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Control of chaos in the single-mode optically pumped far-infrared $^{15}NH_3$ laser is experimentally demonstrated using continuous time-delay control. Both the Lorenz spiral chaos and the detuned period-doubling chaos exhibited by the laser have been controlled. While the laser is in the Lorenz spiral chaos regime the chaos has been controlled both such that the laser output is cw, with corrections of only a fraction of a percent necessary to keep it there, and to period one. The laser has also been controlled while in the period-doubling chaos regime, to both the period-one and -two states.
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There are many systems that are limited in performance because the inherent nonlinearities which they possess induce instability and chaos. This behavior has had to be tolerated with the result of having a reduction in performance. The advent of techniques in the control of chaos may in the future lead to significant enhancement in the performance of many systems.

Pioneering work in the control of temporal chaos has been published in several notable papers [1–5]. The first control of chaos method that was adapted to experiment is known as the OGY method after the three authors Ott, Grebogi, and Yorke [1]. Their method demonstrates that the motion of a chaotic system can be reduced to periodic motion by perturbatively controlling the system about one of the many unstable periodic orbits that exist in the chaotic attractor. This method was used to experimentally control a magnetoelastic ribbon on a period-one and -two orbit [2], and has also been used to control the motion of water in a thermal convection loop [4].

More recently a different method of control has been shown to be successful in the experimental control of chaos. This method involves the continuous generation of an error signal from the difference between the output signal and its value at an earlier time. It is called continuous time-delay feedback and was first demonstrated by Pyragas in 1992 [6]. Subsequently Pyragas and Tamaševičius [7], followed by Bielawski *et al.* [8], have shown it to be a robust method for the control of chaos in electrical circuits and a CO_2 laser with modulated losses, respectively. In the paper by Gauthier *et al.* [9] it was also demonstrated that the method was very useful for the application to fast systems by using it to control a chaotic nonlinear circuit driven at 10.3 MHz. These experimental results all pertain to nonautonomous systems where the chaos results from driving the system at a well-defined frequency. We have now demonstrated control of chaos using continuous time-delay feedback in a fundamentally different system: a single-mode laser that displays Lorenz-like chaos. A nonautonomous system driven to chaos has a sharply defined frequency to which the control can respond. This stability in the phase does not occur for Lorenz-like chaos because the phase is as chaotic as the intensity.

In our experiments the laser pump power is varied in response to a continuous error signal generated by comparing the laser output with its value at an earlier time. A great virtue of this method is that it does not require knowledge of more than one variable. The experimental setup is shown schematically in Fig. 1. The most significant parts of the setup include the $^{13}CO_2$ laser pump and the method whereby its output is modulated. The rest of the setup including the $^{15}NH_3$ laser operating at 153-μm wavelength and Schottky barrier diode detector is the same as has been previously reported by Win *et al.* [10]. The pump power is controlled by the use of an acousto-optic modulator (AOM). The power of the rf traveling wave injected into the crystal determines how much of the incident light is diffracted. Amplitude modulating the RF drive therefore gives a simple means of producing a time varying pump power.

The continuous time delay error signal was generated in the following way. Approximately 90% of the output signal from the Schottky diode detector is fed back into a simple

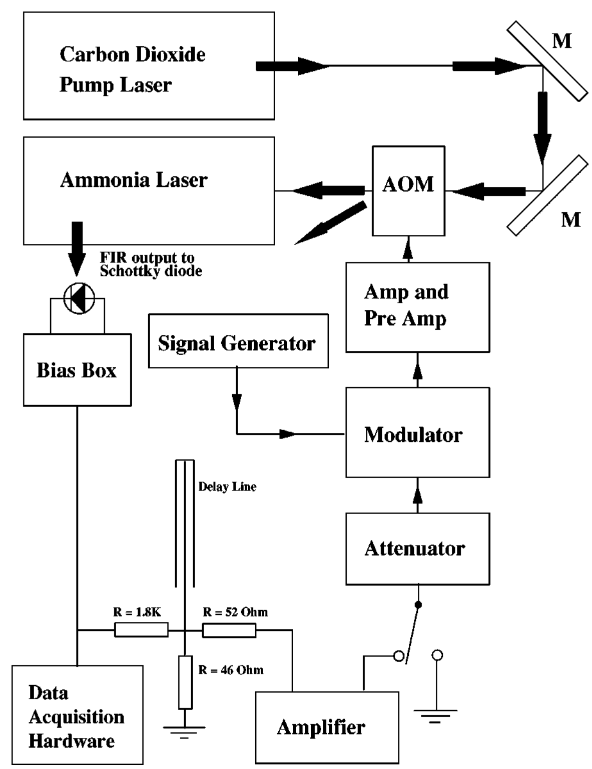

FIG. 1. The experimental setup.

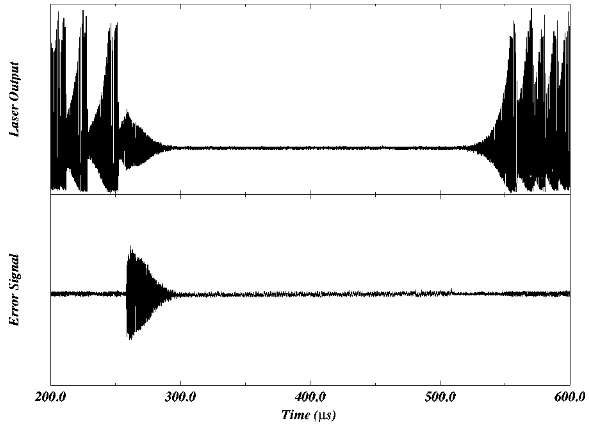

FIG. 2. Control to the fixed point when the laser demonstrates Lorenz chaos with the lower trace showing the error signal and the upper the laser output. The feedback loop is closed at 260 μs and opened at 520 μs.

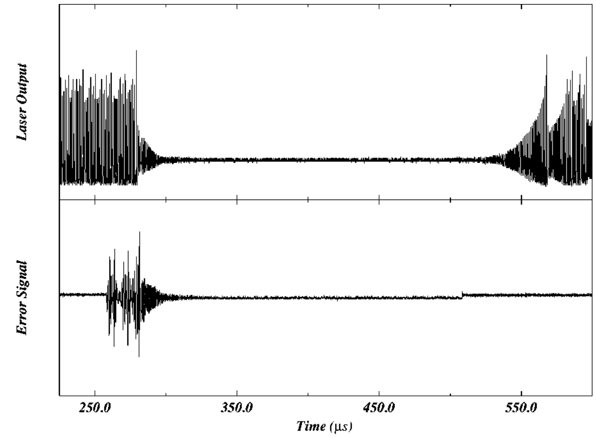

FIG. 3. Control to the fixed point when the laser demonstrates Lorenz chaos with the lower trace representing the error signal and the upper the laser output. The feedback loop is closed at 260 μs and opened at 520 μs.

power splitter recombiner arrangement. The remaining 10% is used to sample the output intensity of the laser. The power splitter recombiner resistors have values such that the reflection losses are minimized and 50% of the incident electrical power is injected into a delay line. That part of the signal that enters the delay line is reflected from the short circuited end and therefore inverted. Upon returning to the junction this signal recombines with that part which arrives at the junction the delay time later and does not enter the delay line. The recombined signal forms an error signal that is then amplified, put through a switch that allows it to be turned on and off, then attenuated if necessary, and finally used to modulate the drive power for the AOM. With the length of cable used (80 m initially to give approximately 0.8 μs delay plus whatever lengths are added) the Ohmic losses were recorded to be about 5–10 %. The time for the error signal to reach the AOM, travel through the crystal, and finally the pump beam requires 2.2 μs. The form of the pump power and hence the error signal with time (from the diffracted beam) is observed using a $Hg_xCd_{1-x}Te$ detector. It should be noted that the feedback of the error signal is always positive unless stated otherwise. By positive we mean that the error signal is not inverted before the pump power is modulated. The effect of the feedback depends on the delay which cannot easily be varied in our experiment. However, the control of chaos in the laser was relatively easy to observe as the frequency of the natural oscillations in the laser changes markedly with small changes in the tuning of the cavity or the pump. The changes in frequency are large enough to allow control to be observed with many different lengths of delay line. This natural variability would make application of this control method a good prospect. Figure 2 shows the laser displaying Lorenz-like chaos. However, once the feedback loop is closed the laser quickly moves onto the fixed point, after which the error signal becomes very small. The maximum change in pump power in this record is only about 3%. The simple form of the transient as the loop gains control is not the most general or usual one as usually, there is a period of metastable chaotic behavior. A more typical case is shown in Fig. 3 where there is some metastability and the maximum peak-to-peak amplitude is 7% of the total pump power. This sort of transient seems to occur no matter what type of dynamics is controlled in the laser. The control signal shown in Fig. 3, once the laser has moved onto the fixed point, is so small that the recorded trace does not have the resolution to show it. In Fig. 2, however, the control signal is possibly visible and of magnitude 0.1% of the total pump power. This control is therefore clearly a perturbative method that stabilizes the fixed points. This could be a robust method for obtaining greater cw powers out of the single mode laser. The control demonstrated in Figs. 2 and 3 is obtained only for limited parameter ranges and for fixed delay line lengths. The most dominant behavior is the control to a period-one pulsation as shown in Fig. 4.

This is not surprising as the control to cw lasing can be thought of as being a special case of the control to the period one. In Lorenz chaos there exist three unstable fixed points in the phase space, one at the origin and two from which the trajectories always spiral away. These spirals increase in period as their distance from the fixed point increases. The control mechanism with the delays incorporated as they are embodies a specific resonance frequency because of the signal's phase. Change the length of the delay line a fraction of

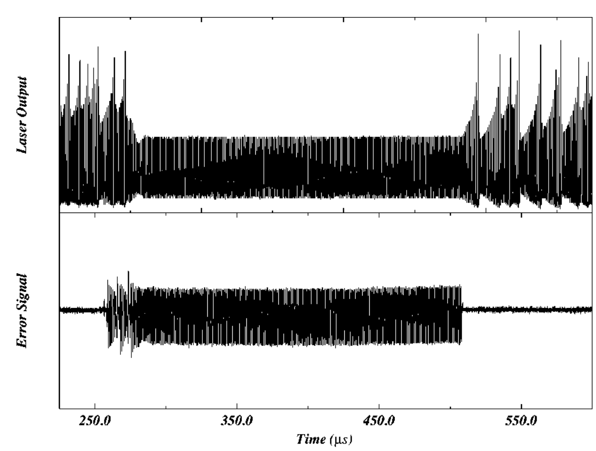

FIG. 4. Control to period one when the laser demonstrates Lorenz chaos with the lower trace representing the error signal and the upper the laser output. The feedback loop is closed at 260 μs and opened at 520 μs.

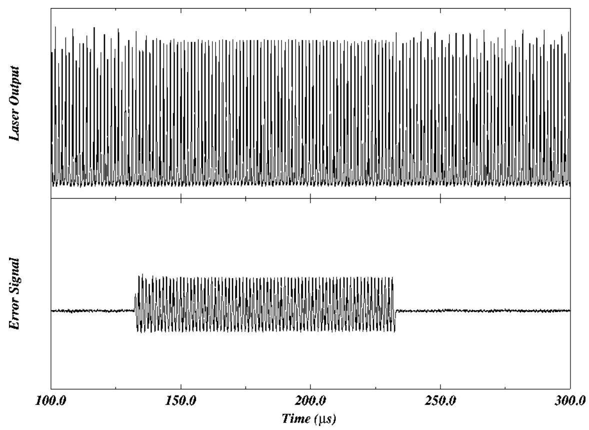

FIG. 5. Control to period one when the laser demonstrates period-doubling chaos with the lower trace representing the error signal and the upper the laser output. The feedback loop is closed at 130 μs and opened at 230 μs.

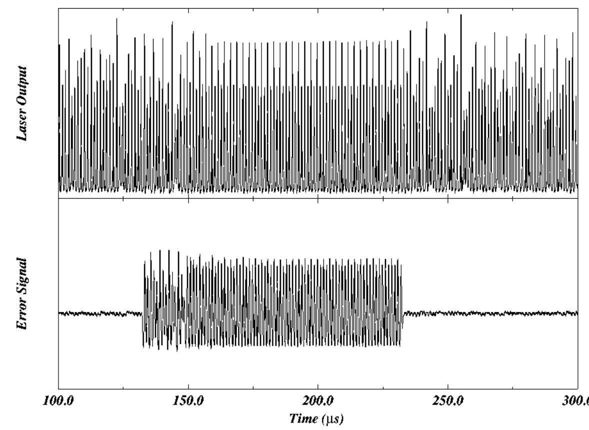

FIG. 6. Control to period two when the laser demonstrates period-doubling chaos with the lower trace representing the error signal and the upper the laser output. The feedback loop is closed at 260 μs and opened at 230 μs.

the natural period of the laser's output intensity and the phase will cause a large change in the error signal. Make the delay equal to the natural period of the laser's output intensity and the error signal is minimized, hence the resonance. Similarly there is another sensitivity in phase with the length of the delay for the signal to be fed back via the pump. Since this delay exceeds the pulsation period, the feedback may be effectively positive or negative depending on the exact timing. There is therefore a characteristic spiral radius and thus period where the orbit is stabilized by the effect of the control. Decreasing, for example, the delay line's length increases the periodic orbit's frequency and thus decreases the radius. This could be done up to the point where the laser seems to emit cw only, with the orbit radius too small to be seen above the noise. As changing the length of the delay line is difficult, a more attractive way to explore this behavior is to change the tuning of the laser slightly such that the Lorenz chaos has a lower characteristic frequency. In this way it was observed that the period-one output intensity amplitude and the error signal amplitude decreased in size as the tuning was changed in such a way that the natural oscillation period of the laser decreased. This can be regarded as the directing of an orbit to a target analogous to the method described by Shinbrot *et al.* [11] in the sense that a trajectory on an orbit can be directed to a desired state in predetermined way.

The control exhibited to the period one is not really perturbative as the error signal has a peak-to-peak amplitude of approximately 5% of the pump power. Nonperturbative control has also been observed while the laser is detuned away from Lorenz spiral chaos, in single mode period-doubling chaos. Figures 5 and 6 demonstrate the control with error signals of peak-to-peak amplitude, approximately 5% of the pump power. In period-doubling chaos the periodic orbits (i.e., period one, two, three, etc.) exist very close to each other in phase space. The periods of the orbits divided by their order are however never quite equal. This thereby allows the resonance of the control to pick out individual periodic orbits of different order by making small changes in the tuning of the laser in the same manner as described in the paragraph above. Figure 5 shows the stabilization to a period-one orbit and Fig. 6 shows the stabilization to a period-two orbit, having used only a small amount of tuning to get from one situation to the other.

In conclusion, we have demonstrated control of chaos by using the time delay feedback method on a single mode laser that displays Lorenz-like chaos. We have demonstrated control to cw lasing and a period-one orbit when the laser displays Lorenz-like chaos. It has also been shown that when the laser is detuned to exhibit period-doubling chaos control is also possible with period-one and period-two orbits, although not perturbatively.

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